

Second year Higher Secondary Examination**PART III****MATHEMATICS (SCIENCE)**

Maximum: 80 (Scores)

TIME: $2\frac{1}{2}$ Hours

Cool-off time: 15 minutes

GENERAL INSTRUCTIONS TO CANDIDATES:

- There is a 'Cool-off time' of 15 minutes in addition to the writing time of $2\frac{1}{2}$ hours.
- You are not allowed to write your answers or to discuss anything with others during the 'Cool-off time'.
- Use 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

Questions 1 to 7 carry 3 score each. Answer any six.

- Set A has 3 elements and set B has 4 elements. Then the number of injective mappings that can be defined from A to B is
 i) 144 ii) 12 iii) 24 iv) 64 (1)
 - If $f : R \rightarrow R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $f \circ f(x)$ is (2)
- If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then prove that $A^2 - 4A + 7I = 0$. Also find A^{-1} . (3)
- Determine if f defined by $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is a continuous function? (2)
 - For the non-singular matrix A, $(A^{-1})' = \dots\dots\dots$
 (a) $(A^{-1})^{-1}$ (b) $(A')^{-1}$ (c) A (d) None of these (1)
- Integrate:
 a) $\int_{-a}^a f(x) dx = 0$, if $f(x)$ is
 a) an even function b) an odd function
 c) identity function d) None of these (1)

$$b) \int \sqrt{\frac{1+x}{1-x}} dx \quad (2)$$

$$5. \text{ Evaluate } \int_0^2 e^x dx \text{ as the limit of a sum.} \quad (3)$$

$$6. \text{ Solve the differential equation: } \cos\left(\frac{dy}{dx}\right) = a \ (a \in R); y = 1, \text{ when } x = 0. \quad (3)$$

$$7. a) \text{ If } |\vec{a}| = \sqrt{3}, |\vec{b}| = 2 \text{ and angle between } \vec{a} \text{ and } \vec{b} \text{ is } 60^\circ, \text{ find } \vec{a} \cdot \vec{b} \quad (1)$$

$$b) \text{ Find a unit vector perpendicular to each of the vectors } \vec{a} + \vec{b} \text{ and } \vec{a} - \vec{b}, \text{ where } \vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k} \quad (2)$$

Questions 8 to 17 carry 4 score each. Answer any eight.

8. a) Consider a binary operation $*$ on the set $\{1,2,3,4,5\}$ given by the following multiplication table
- i) Compute $(2*3)*4$ and $2*(3*4)$ (1)
- ii) Is $*$ commutative? (1)

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

$$b) \text{ Show that the function } f : R \rightarrow R, \text{ defined as } f(x) = 3x, \text{ is bijective? If so, find } f^{-1}. \quad (2)$$

$$9. a) \text{ The value of } \tan^{-1}(-\sqrt{3}) \text{ is} \quad (1)$$

$$b) \text{ Write the simplest form of } \tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{\pi}{2} < x < \frac{3\pi}{2} \quad (3)$$

$$10. a) \text{ The function } f(x) = [x], \text{ where } [x] \text{ denotes the greatest integer function, is continuous at} \\ \text{A) 3} \quad \text{B) -4} \quad \text{C) 1} \quad \text{D) 1.5} \quad (1)$$

$$b) \text{ Find the values of a and b such that the function defined by} \\ f(x) = \begin{cases} 5, & \text{if } x \leq 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \geq 10 \end{cases} \text{ is a continuous function.} \quad (3)$$

$$11. a) \text{ For the curve } y = 5x - 2x^3, \text{ if } x \text{ increases at the rate of 2 units/sec, then how fast is the} \\ \text{slope of the curve changing when } x = 3? \quad (2)$$

$$b) \text{ A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground,} \\ \text{away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the} \\ \text{foot of the ladder is 4m away from the wall?} \quad (2)$$

$$12. \text{ Evaluate } \int \frac{\sin x}{(1-\cos x)(2-\cos x)} dx \quad (4)$$

$$13. a) \text{ Form the differential equation representing the family of curves: } y = a \cos(x+b), \text{ where A} \\ \text{and b are constants.} \quad (2)$$

b) Solve: $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1+x^2}} = 0$. (2)

14. a) For what value of λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other? (2)

b) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 5, |\vec{b}| = 12$ and $|\vec{c}| = 13$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}$. (2)

15. a) Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, -4), (-1, 1, 2)$ and $(-5, -5, -2)$. (2)

b) Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the XY plane. (2)

16. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point $P(3, 2, 1)$ from the plane $2x - y + z + 1 = 0$. Find also the image of the point in the plane. (4)

17. Find the particular solution of the following differential equation:

$$\frac{dy}{dx} + x \cot y = 2y + y^2 \cot y, \quad y \neq 0, \text{ given that } x = 0 \text{ when } y = \frac{\pi}{2}. \quad (4)$$

Questions 18 to 24 carry 6 score each. Answer any Five questions.

18. a) If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

A) A B) $I - A$ C) $I + A$ D) $3A$ (1)

b) Find the values of x and y from the following equation:

$$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix} \quad (2)$$

c) If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$. (3)

19. Let A be a nonsingular square matrix of order 3×3 . Then $|adj A|$ is equal to

a) $|A|$ b) $|A|^2$ c) $|A|^3$ d) $3|A|$ (1)

b) Using the properties of determinants, prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3 \quad (5)$$

20. Find $\frac{dy}{dx}$:

a) $y = \sqrt{\tan \sqrt{x}}$ (2)

b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$ (2)

c) $x = ct$; $y = \frac{c}{t}$ (2)

21. Find the following integrals:

a) $\int_0^{2a} f(x) dx = 0$, if $f(2a - x) = \dots\dots\dots$ (1)

b) $\int \sqrt{x^2 + 4x + 1} dx$ (2)

c) $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ (2)

22. a) The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is (1)

A) 20π sq units B) $20\pi^2$ sq units

C) $16\pi^2$ sq units D) 25π sq units

b) Using integration find the area of the region in the first quadrant enclosed by the x-axis line

$x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$. (5)

23. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit, of Rs 17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day?

a) Write the objective function. (1)

b) Write the constraints. (1)

c) Find the feasible region. (2)

d) Find the corner points (1)

e) Find the maximum profit. (1)

24. a) An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question? (3)

b) A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes. (3)

=====