125

3rd CiviL

Hydraulics

Open Channel

Chapter 2

Circular Section

V_{max} and Q_{max}

No. 09

Problem (11) Sh≠2

Sheet غير 1:600

$S = 1/5000$

$n = 0.013$

$D = 1\ m$

- **Required**

1- Draw Relation between $\frac{y}{D}$ & $\left(\frac{Q}{Q_{full}} \text{ \& } \frac{V}{V_{full}}\right)$

From the Curves obtain the value $\frac{y}{D}$ for V_{max} & Q_{max}

2- Analytically لـ find the depths of V_{max} & Q_{max} using maning Eq.

3- Analytically find the depths of V_{max} & Q_{max} Using chezy Eq.

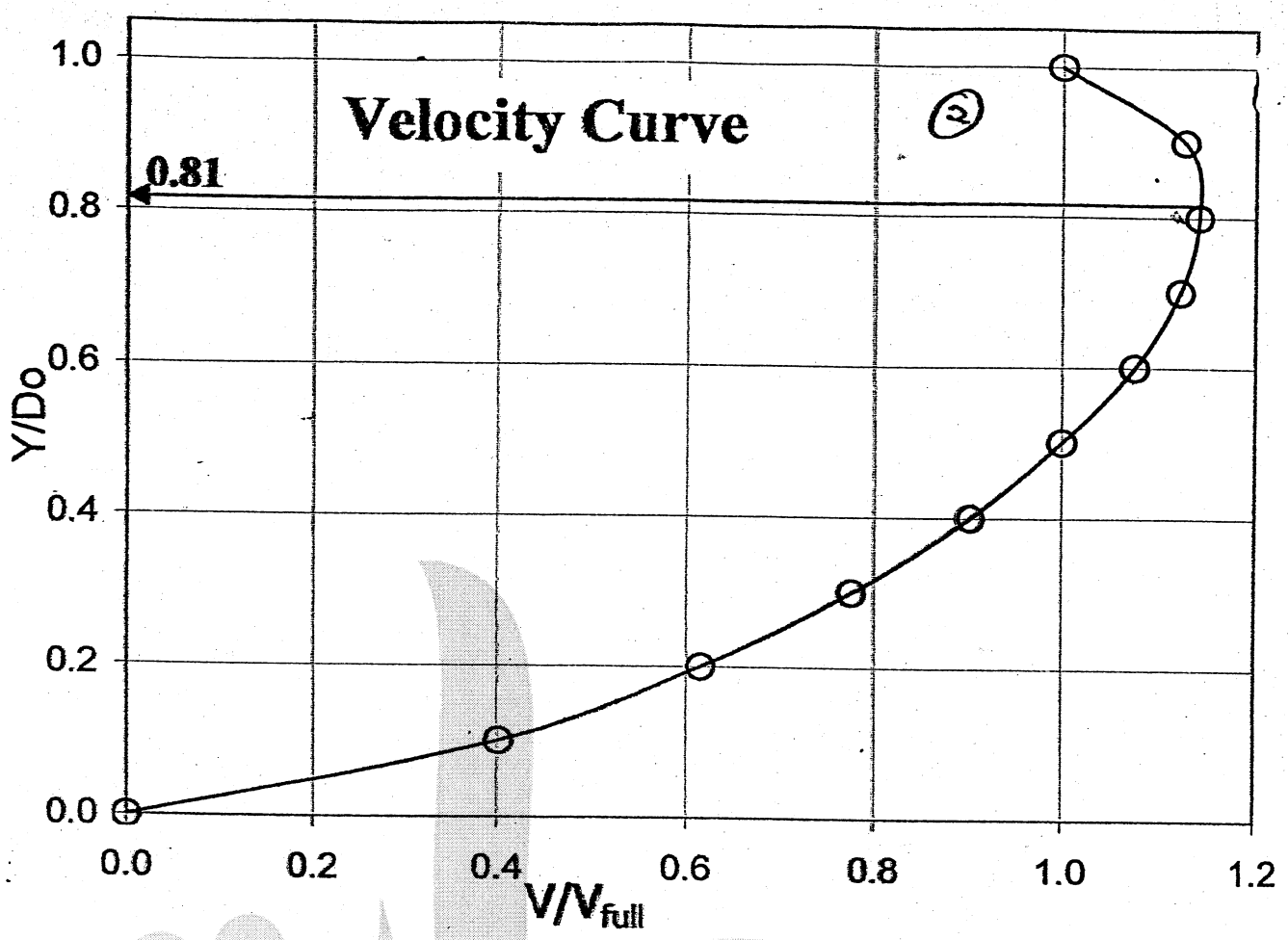
- **SoL**

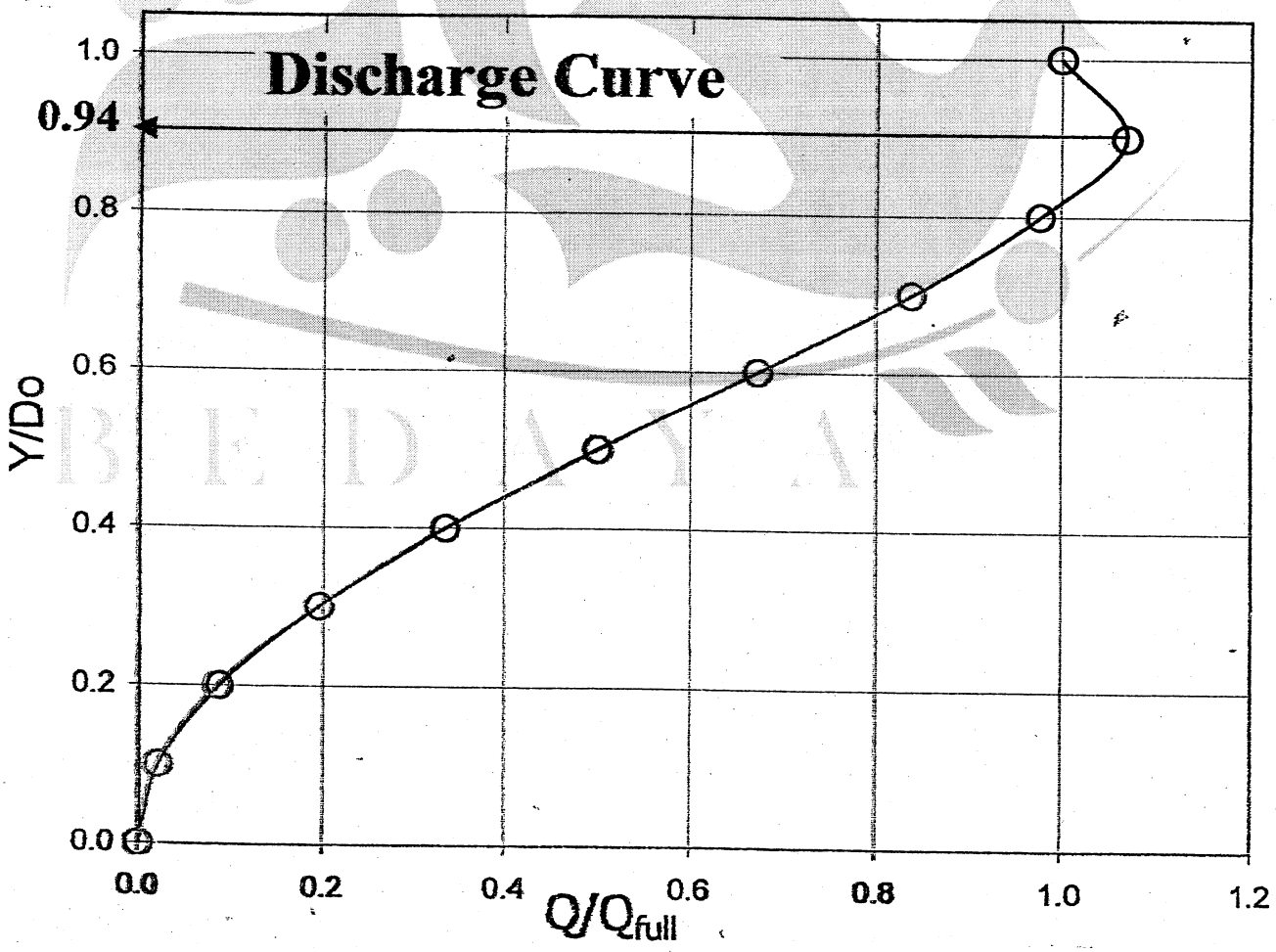
نفرض قيم $\frac{y}{D}$ متغيره من صفر (Empty pipe) لـ 1 (full pipe)

وحسب مقاطع y ، A ، P ، R ، V ، Q لكل قيمة ونوجد نسب $\frac{Q}{Q_{full}}$ ، $\frac{V}{V_{full}}$

ونجد القطاع ممتلئ بالكامل

y/D	y	θ	θ°	A	P	R	V	Q	V/Vfull	Q/Q
0	0	0	0	0	0	0	0	0	0	0
0.1	0.10	1.29	73.74	0.04	0.64	0.06	0.17	0.01	0.40	0.0
0.2	0.20	1.85	106.26	0.11	0.93	0.12	0.27	0.03	0.62	0.0
0.3	0.30	2.32	132.84	0.20	1.16	0.17	0.34	0.07	0.78	0.2
0.4	0.40	2.74	156.93	0.29	1.37	0.21	0.39	0.11	0.90	0.3
0.5	0.5	3.14	180.00	0.39	1.57	0.25	0.43	0.17	1	0.
0.6	0.60	3.54	203.07	0.49	1.77	0.28	0.46	0.23	1.07	0.6
0.7	0.70	3.96	227.16	0.59	1.98	0.30	0.48	0.28	1.12	0.8
0.8	0.80	4.43	253.74	0.67	2.21	0.30	0.49	0.33	1.14	0.9
0.9	0.90	5.00	286.26	0.74	2.50	0.30	0.49	0.36	1.12	1.0
1	1.0	6.28	360	0.79	3.14	0.25	0.43	0.34	1	1


Velocity Curve
0.81
Y/Do
V/Vfull
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2


Discharge Curve
0.94
Y/Do
Q/Qfull
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2

Req # 3 Check your result using Chezy

[A] For maxi. Velocity

$$V = C\sqrt{RS} = C S^{1/2} R^{1/2}$$

$V = V_{max}$ at $R = R_{max}$

$$R = \frac{A}{P} = \frac{d}{4}\left(\frac{\theta_r - \sin\theta}{\theta_r}\right) = \frac{d}{4}\left[1 - \frac{\sin\theta}{\theta_r}\right]$$

$R = R_{max}$ at $\frac{dR}{d\theta} = 0.0$

$$\frac{d}{4}\left[0 - \frac{\theta_r \cos\theta - \sin\theta}{\theta_r^2}\right] = 0.0$$

$$0,0 - \frac{d}{4}\frac{\theta_r \cos\theta - \sin\theta}{\theta_r^2} = 0.0$$

$$\theta_r \cos\theta - \sin\theta = 0.0$$

$$\theta_r = \tan\theta$$

by trial $\longrightarrow$ $\theta = 257,5^\circ$

$$y = \frac{D}{2}\left[1 - \cos\frac{\theta}{2}\right]$$

$$y = \frac{D}{2}\left[1 - \cos\frac{257,5}{2}\right]$$

$$y = 0.81\,D$$

∴ Maximum Velocity at $\theta = 257,5^\circ$, $y = 0.81\,D$

[B] For maxi. Discharge

$$Q = CA\sqrt{RS} = C\sqrt{S}\,\frac{A^{3/2}}{P^{1/2}}$$

$$Q = Q_{max} \text{ at } \frac{d}{d\theta}\left[\frac{A^{3/2}}{P^{1/2}}\right] = 0.0$$

$$\frac{\frac{3}{2}P^{1/2}A^{1/2}\frac{dA}{d\theta} - \frac{1}{2}A^{3/2}P^{-1/2}\frac{dP}{d\theta}}{P} = 0.0$$

$$\frac{3}{2}P^{1/2}A^{1/2}\frac{dA}{d\theta} = \frac{1}{2}A^{3/2}P^{-1/2}\frac{dP}{d\theta}$$

$$3P\frac{dA}{d\theta} = A\frac{dP}{d\theta}$$

$$\frac{dA}{d\theta} = \frac{d}{d\theta}\left[\frac{d^2}{8}(\theta_r - \sin\theta)\right] = \frac{d^2}{8}(1-\cos\theta)$$

$$\frac{dP}{d\theta} = \frac{d}{d\theta}\left[\frac{d}{2}\theta_r\right] = \frac{d}{2}$$

$$3 * \frac{d}{2}\theta_r * \frac{d^2}{8}(1-\cos\theta) = \frac{d^2}{8}(\theta_r - \sin\theta)\cdot\frac{d}{2}$$

$$3(\theta_r - \theta_r\cos\theta) = \theta_r - \sin\theta$$

$$3(\theta_r - \cos\theta\cdot\theta_r) - \theta_r + \sin\theta = 0.0$$

by trial $\longrightarrow$ $\theta = 308,16^{\circ}$

$$y = \frac{d}{2}[1-\cos\theta] = \frac{d}{2}\left[1-\cos\frac{308,16^{\circ}}{2}\right]$$

$$y = 0.95d$$

(7)

- if required or given **Max. velocity**

  - **manning** $y = 0.81D$, $\theta = 257,5°$
  - **chezy** $y = 0.81D$, $\theta = 257,5°$

- if required or given **Max. discharge**

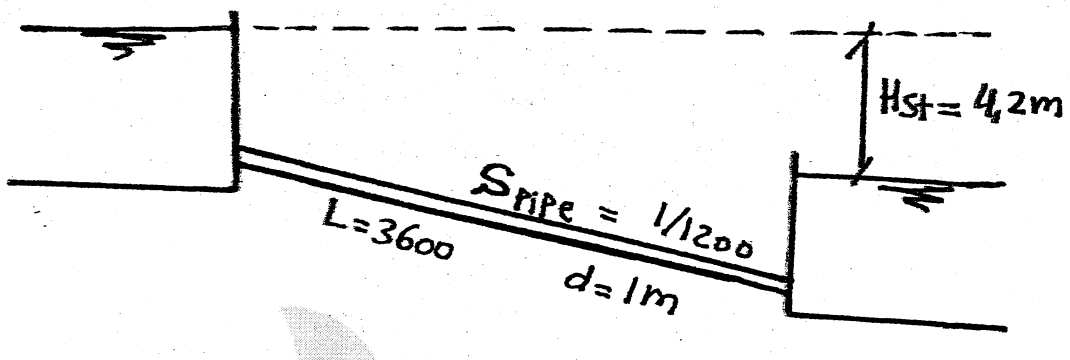
  - **manning** $y = 0.94D$, $\theta = 302,4°$
  - **chezy** $y = 0.95D$, $\theta = 308,16°$

Also if required "τ_{max}" $= \gamma RS$

$\therefore \tau_{max}$ at R_{max} like V.

So "τ_{max}" $\longrightarrow$ $y = 0.81D$, $\theta = 257,5°$

Problem (16) Sh#2



Given $Q = 0.525\ m^3/s$ (Running full)

$C = K R^{1/6}$

Req

1- n, C, K

2- at $Q = 0.325$ $y = ?$ f, n, C

Solution

لم يتم اعطاء n أو C ∴ لا يوجد غير معادلة الفواقد

$$H_{st} = h_f = \frac{fL}{4R} * \frac{V^2}{2g}$$

$$V = \frac{Q}{A} = \frac{0.525}{\pi/4 * 1^2} = 0.668\ m/s$$

Full Pipe ∴ $R = \frac{d}{4} = \frac{1}{4} = 0.25$

∴ $4.2 = \frac{f * 3600}{4 * 0.25} * \frac{0.668^2}{2g}$ ∴ $\boxed{f = 0.05123}$

$\therefore C = \sqrt{\frac{8g}{f}} = \sqrt{\frac{8g}{0.05123}}$ $\therefore$ **$C = 39,14 \ m^{1/2}/s$**

$C = K R^{1/6}$ $\therefore$ **$R = 49.32$**

But $C = \frac{1}{n} R^{1/6}$ $\therefore K = \frac{1}{n}$ $\therefore$ **$n = 0.0203$**

[B]

Partially full حالة و بالتالى تصبح Q

- $A = \frac{D^2}{8}(\theta_r - \sin\theta) = 0.125(\theta_r - \sin\theta)$
- $P = d/2\,\theta_r = 0.5\theta_r$

Manning

$$0.325 = \frac{1}{0.0203} * \frac{[0.125(\theta_r - \sin\theta)]^{5/3}}{[0.5\theta_r]^{2/3}} * \sqrt{1/1200}$$

By trial : **$\theta = 210^\circ$**

- $y = \frac{D}{2}[1 - \cos\frac{\theta}{2}] = 0.63\,m$

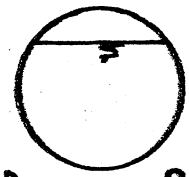
$$R = \frac{A}{P} = \frac{1^2/8(210 * \frac{\pi}{180} - \sin 210)}{210 * \frac{\pi}{180} * \frac{1}{2}} = 0.28\,m$$

- $C = \frac{1}{n} * R^{1/6} = \frac{1}{0.0203} * 0.28^{1/6} = \underline{39,95} \ m^{1/2}/s$

$C = \sqrt{\frac{8g}{f}}$ $\quad 39,95 = \sqrt{\frac{8g}{f}}$

$\therefore$ **$f = 0.049$**

Problem (12) Sh#2 (final exam 2003)



Given:- $S = 0.00062$

$Q = 120 * 10^{-3} = 0.12$

$V = 0.6$

$n = 0.013$

Req:-

1- diameter

2- Q_{max}

3- For half full pipe $E, F_n, Z = ?$

4- Sketch relation betw. y & Z

Sol:-

$$A = \frac{Q}{V} = \frac{0.12}{0.6} = 0.2$$

$$d^2/8 \ (\theta_r - \sin\theta) = 0.2 \longrightarrow [1]$$

$$Q = \frac{1}{n} * \frac{A^{5/3}}{P^{2/3}} * S^{1/2}$$

$$0.12 = \frac{1}{0.013} * \frac{0.2^{5/3}}{P^{2/3}} * 0.00062^{0.5}$$

$$P = 1.14$$

$$d/2 * \theta_r = 1.14 \qquad d = \frac{2.28}{\theta} \longrightarrow [2]$$

بالتعويض من (2) في (1)

$$\left(\frac{2.28}{\theta_r}\right)^2/8 * (\theta_r - \sin\theta) = 0.2$$

by trial $\theta = 213,7^\circ$

$$\therefore \boxed{d = 0.61\ m}$$

2- For Qmax $\theta = 302,4^\circ$

manning

$$Q_{max} = \frac{1}{0.013} * \frac{\left[\frac{0.61^2}{8}(\theta_r - \sin\theta)\right]^{5/3}}{\left[\frac{\theta_r}{2} * 0.61\right]^{2/3}} * \sqrt{0.00062}$$

$$Q_{max} = 0.172 \quad m^3/s$$

3- half-full Pipe

$$\theta = 180^\circ,\ y = \frac{D}{2},\ R = \frac{D}{4} = 0.153$$

manning

$$V = \frac{1}{0.013} * 0.153^{2/3} * 0.00062^{0.5} = 0.548\ m/s$$

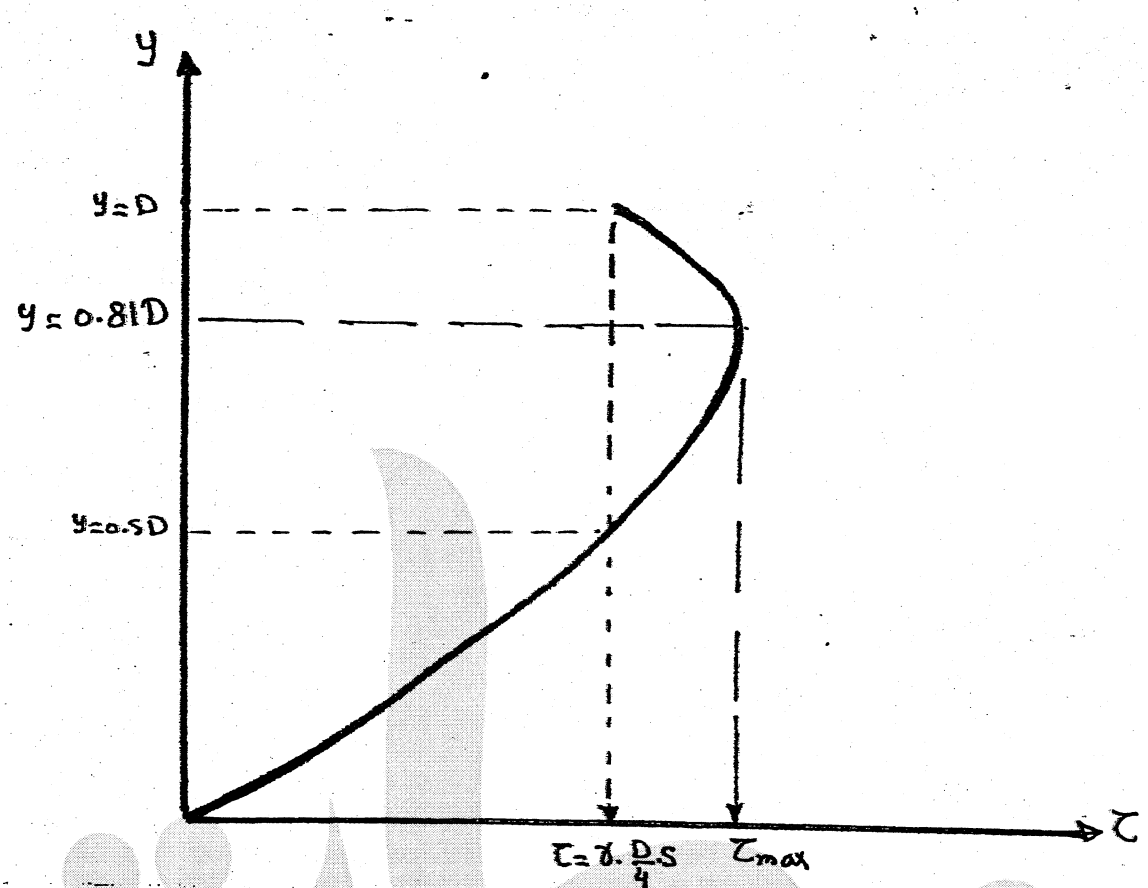
$$E = y + \frac{V^2}{2g} = \frac{0.61}{2} + \frac{0.548^2}{2g} = 0.32\ m$$ ← طاقة بالماء

$$Fr = \frac{V}{\sqrt{g\,y_h}} = \frac{0.548}{\sqrt{g * 0.24}} = 0.357$$

$$y_h = \frac{A}{T} = \frac{1/2\ \pi/4 * 0.61^2}{0.61} = 0.24\ m$$

$$\tau = \gamma R S = 1 * 0.153 * 0.00062 = 94.86 * 10^{-6}\ t/m^2$$

[d] Relation between τ & y



● ملاحظات على المنحنى

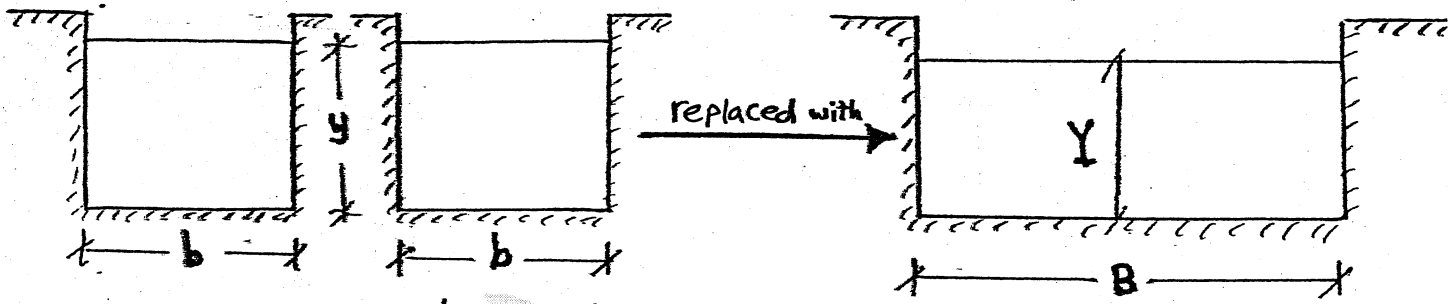
1- أقصى قيمة τ_{max} عندما $y = 0.81D$ هي V_{max}

2- قيمة τ متساوية عند $y = \frac{D}{2}$ & $y = D$
half-full full

لأنه في الحالتين $R = \frac{D}{4}$

$$\tau = \gamma R S$$

Problem (14). Sh≠ (2)



Two identical channels
قناتان

one channel

Given

- Same n, S
- $\frac{y}{b} = 0.3$, $\frac{Y}{B} = 0.3$

Required

Relative Saving in Cost of excavation ?

Sol.

ترعتان متشابهتان يراد استبدالهما بترعة واحدة لإمرار نفس التصرف المطلوب حساب نسبة التوفير في حجم الحفر الكلي للترع

$$2\, Q_{\square\, b\times y} = Q_{\square\, B\times Y}$$

$$2 \frac{1}{n} * \frac{A_2^{5/3}}{P_2^{2/3}} * S^{1/2} = \frac{1}{n} \frac{A_1^{5/3}}{P_1^{2/3}} * S^{1/2}$$

$$2 \frac{A_2^{5/3}}{P_2^{2/3}} = \frac{A_1^{5/3}}{P_1^{2/3}}$$

$$2 * \frac{(by)^{5/3}}{(b+2y)^{2/3}} = \frac{(BY)^{5/3}}{[B+2Y]^{2/3}}$$

But: $y = 0.3b$, $Y = 0.3B$

$$\therefore 2 * \frac{(0.3b^2)^{5/3}}{(1.6b)^{2/3}} = \frac{(0.3B^2)^{5/3}}{(1.6B)^{2/3}}$$

$$2 * \frac{(0.3 b^2)^{5/3}}{(1/6 b)^{2/3}} = \frac{(0.3 B^2)^{5/3}}{(1/6 B)^{2/3}}$$

$$2 * \frac{b^{10/3}}{b^{2/3}} = \frac{B^{10/3}}{B^{2/3}}$$

$$2 * b^{8/3} = B^{8/3}$$

$$B = 1.297\, b \simeq 1.3\, b$$

- Relative Saving in Cost of excavation

$$= \frac{2a - A}{2a} \times 100$$

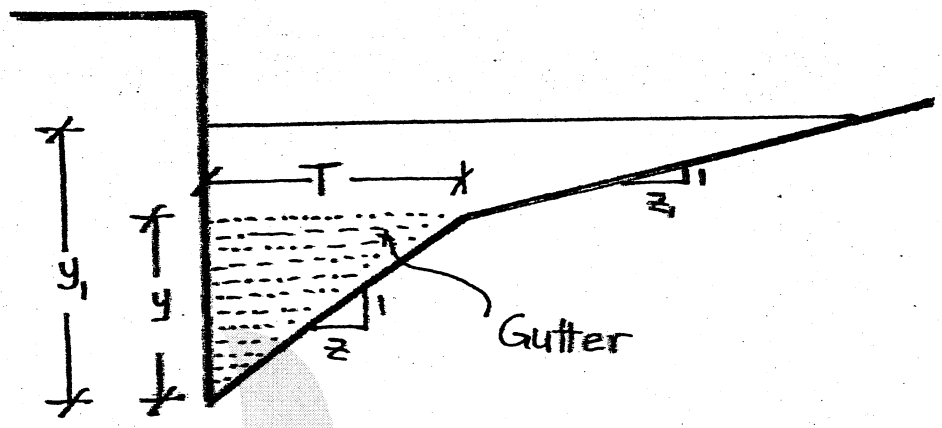
$$= \frac{2(0.30 b^3) - 0.30 B^2}{2(0.30 b^2)} \times 100$$

$$= \frac{0.6 b^2 - 0.30 (1.3b)^2}{0.6 b^2} \times 100$$

$$= \frac{0.093 b^2}{0.6 b^2} \times 100$$

$$= 15,5\ \%$$ #

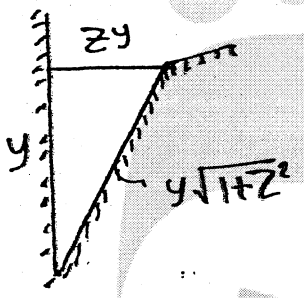
Problem (4) Sh#2



A) Prove that :-

Gutter discharge $Q = \frac{0.315}{n} * \frac{z^{5/3}}{(1+\sqrt{1+z^2})^{2/3}} * y^{8/3} * S^{1/2}$

Sol:-



$A = y * T/2 = y * zy/2 = zy^2/2$

$P = y + y\sqrt{1+z^2}$

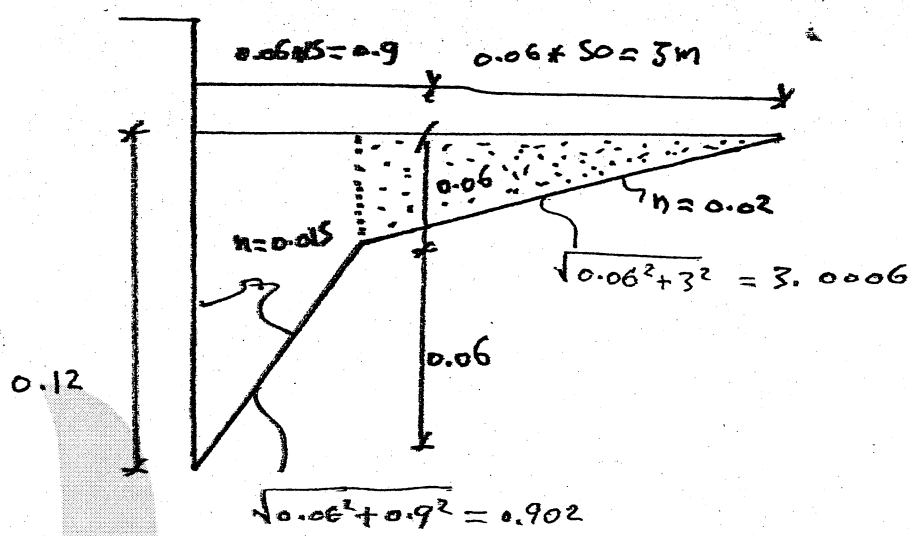
$Q = \frac{1}{n} * \frac{A^{5/3}}{P^{2/3}} * S^{1/2}$

$= \frac{1}{n} * \frac{(zy^2/2)^{5/3}}{(y+y\sqrt{1+z^2})^{2/3}} * S^{1/2}$

$Q = \frac{1}{n} * (\frac{1}{2})^{5/3} * \frac{z^{5/3} \; y^{10/3}}{y^{2/3} \; (1+\sqrt{1+z^2})^{2/3}} \cdot S^{1/2}$

$Q = \frac{0.315}{n} * \frac{z^{5/3}}{(1+\sqrt{1+z^2})^{2/3}} * y^{8/3} * S^{1/2}$ #

العلاقة المطلوبة



$n_1 = 0.015$

$P_1 = 0.12 + 0.902 = 1.022$ m

$n_2 = 0.02$

$P_2 = 3.0006$ m

$P = P_1 + P_2 = 4.023$ m

Horton

$$n_e = \frac{\left[1.022 * 0.015^{1.5} + 3.0006^{1.5}\right]^{2/3}}{4.023}$$

$n_e = 0.0188$

$$Q = \frac{1}{n_e} \frac{A^{5/3}}{P^{2/3}} \sqrt{S}$$

$A = 0.9 * 0.06 + \frac{1}{2} * 0.06 * 0.9 + \frac{1}{2} * 0.06 * 3 = 0.171$ m²

$$Q = \frac{1}{0.0188} * \frac{0.171^{5/3}}{4.023^{2/3}} * \sqrt{8 \times 10^{-3}}$$

$$\boxed{Q = 0.099 \text{ m}^3/\text{s}}$$